EXTRATROPICAL CYCLONES

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Introductory. With the consent of the editor the present article has been written as a summary of the research on Extratropical Cyclones in which the author himself has been directly involved. References to the work of others are therefore few, and the reader will not get any complete survey of the title subject.

The article is built up of two parts. In the first the extratropical cyclones are treated as simplified models for the sake of clarifying the theoretical principles. In the second those principles are applied to a real storm over North America whose development may be considered as the prototype of a simple life history of extratropical cyclones. The modifications of that life history, caused by the varying initial conditions and the influence of neighboring systems in the general circulation, are treated by Dr. E. Palmén in his contribution to the Compendium.

1. DYNAMICS OF SIMPLIFIED CYCLONE MODELS.

a. Theory of pressure changes and thermal structure of extratropical cyclones. The fully developed extratropical cyclone consists of a counterclockwise*) vortex which extends upward into a wave trough in the upper westerlies. The dynamics of the extratropical cyclone therefore is a composite one, combining the dynamic phenomena of the vortex and the wave. We will here state separately the essential features of the atmospheric vortex and the atmospheric wave and then proceed to describing the composite dynamics of the extratropical cyclone.

*) All references to sense of rotation in this article apply to the northern hemisphere.

In analyzing the displacement, intensification and weakening of vortices and waves it is useful to consider the accompanying pressure changes, which obey the "tendency equation"

$$\left(\frac{\partial p}{\partial t}\right)_h = -\int_h^\infty g \, div_H(\rho V)\, dz + (g \, \rho \, v_z)_h \qquad (1)$$

Expressed in words: the rate of pressure change with time in a fixed point at the level h is made up partly by the net horizontal inflow into the vertical unit air column from h to the top of the atmosphere and partly by the vertical inflow of air through the base of that column.

A circular cyclonic vortex with vertical axis centered at the pole of a planet without mountains represents the simplest case of atmospheric vortex dynamics. In the case of frictionless motion in such a polar vortex the particles could be kept in steady state zonal motion from west to east. The horizontal divergence is then everywhere zero and no vertical motion occurs, so that the tendency equation must indicate zero local pressure change at all points. With friction against the ground the flow in the lowest part of the atmosphere would get a component of indraft towards the vortex center, and that horizontal convergence of mass would make the pressure rise in the central portion of the pressure minimum, so that the zonal air motion must slow down. No steady state would be reached until the flow at the ground and the horizontal pressure gradient at the ground have reached zero. If the central core of the vortex is colder than its environment, there would still be a pressure gradient towards the pole in the free atmosphere, and the air may there continue its west to east circulation without horizontal divergence. This picture rather well corresponds to reality as represented by the time-averaged motion in the arctic region: almost zero meridional pressure gradient and zero zonal motion at the ground, and increasing poleward pressure gradient with height, accompanied by increasing westerlies with height. The initial assumption of a cyclone at the pole, and the additional assumption of friction at the ground, thus leads to the dynamical prediction that the cyclone at the ground should eventually disappear, while in the free atmosphere it should be conserved. This behavior of the circular cyclonic vortex can be generalized to apply also at other latitudes, the simple circular vortex is liable to die out gradually at the ground due to friction.

The circular cyclonic vortex centered in middle latitudes does not represent a steady state dynamic system even in case of no friction at the ground. Even though the horizontal pressure gradient may everywhere be directed towards the center and its intensity may be a function only of the distance from the center, the motion around the center cannot be a simple circular one, because the Coriolis factor varies from the northern to the southern part of the vortex. Due to the variation of the Coriolis factor the wind will be stronger in the southern than in the northern part of the vortex, and the net air transport across a north-south median wall will go from the western to the eastern half of the vortex. Consequently the pressure will rise in the eastern half due to horizontal convergence and fall due to horizontal divergence in the western half of the low pressure system, so that the pressure minimum and the accompanying vortex will drift westward. Eccentricity of the pressure field of such sense as to involve a stronger pressure gradient in the southern than in the northern part of the vortex may reduce, neutralize, or even reverse that drift. The dynamic theory for the eccentric vortex has been developed in approximate form by J. Holmboe ( ). He defined a "critical eccentricity" which would balance the exchange of air between the two halves of the vortex. The table of critical eccentricity follows.

Table I

of the quantity $|v|-|v'|-2c = 4\Omega a \sigma_p^2 \cos\varphi$ evaluated in a narrow isobaric channel of critical eccentricity (v and v' wind velocities respectively at southernmost and northernmost point of the isobaric channel, c eastward speed of displacement of the vortex, Ω angular speed of earth, a earth's radius, σ_p angular radius of isobaric channel, φ geographic latitude.

φ	Angular radius of isobaric channel			
	1°	5°	10°	20°
90°	0	0	0	0
80°	0.1 m/sec	2.5	9.9	
70°	0.2	4.9	19.4	78
60°	0.3	7.1	28.4	114
50°	0.4	9.1	36.6	146
40°	0.4	10.9	43.6	175
30°	0.5	12.3	49.4	198

In case of the stationary vortex the exchange of air between the eastern and western halves of the vortex is balanced if the wind velocity in the southernmost point of the isobaric ring exceeds that in the northernmost point by the tabulated amount. Near the center the flow can be almost constant all around the isobaric ring, but the greater the radius the more will the west wind in the south have to exceed the east wind in the north, particularly so in low latitudes.

In applying the table to a moving vortex the double speed of displacement of the vortex must be added to the tabulated speed to give $|v|-|v'|$. For the eastward moving vortex the critical eccentricity is thus stronger than for the stationary one. Even a little more eccentricity would be needed to bring about accumulation of air in the western and depletion of air in the eastern half of the pressure minimum, which would seem necessary to make the system move eastward. A check with measured eccentricities shows, however, only "subcritical" cases; in other words all observed cyclonic vortices accumulate air in their front parts at the expense of the rear parts. The pressure change in such a moving vortex can therefore only be explained if other flow patterns prevail above the vortex. That conclusion is corrobborated by the experience of synoptic aerology, and the typical upper flow pattern above moving vortices is that of the atmospheric wave. In the composite extratropical cyclone the vortex part resists the eastward motion by piling up air in the front half, and this resistance increases the faster the vortex is forced to move.

The atmospheric wave*) superimposes a quasihorizontal oscillation

*) The fundamental properties of the atmospheric waves of synoptic meteorology were first developed by J. Bjerknes ( ), C. G. Rossby (and), and B. Haurwitz ( ). In this article we follow the more recent treatment by J. Bjerknes and J. Holmboe ( ).

upon the fundamental current of straight westerlies. It is accompanied by a periodic distribution of horizontal mass divergence, which in first approximation depends on the relative strength of the "curvature and latitude effects" upon the wind speed. The curvature effect makes the air move supergeostrophically while overtaking the wave crest and subgeostrophically while overtaking the wave troughs. The latitude effect upon the wind speed comes from the fact that each flow channel is in a higher latitude at the anticyclonic bend than at the cyclonic bend, so that, the horizontal pressure gradients being equal, the geostrophic wind would be stronger at the cyclonic than at the anticyclonic bend. The joint result of these two opposite effects on horizontal divergence is in favor of the curvature effect when wave-lengths are short and the fundamental current is strong. In that case the wave crests are preceded by horizontal convergence and the wave troughs by horizontal divergence. In case of long waves and/or a weak fundamental current the opposite distribution of horizontal divergence establishes. That case also represents the conditions found in air layers which move slower eastward than the wave.

In the usual baroclinic westerly current of middle latitudes a wave extending from the slow moving lower layers to the fast moving upper layers would have opposite patterns of horizontal divergence in the upper and lower part (see Figure 1). At the level of transition between the upper and lower pattern a wave motion with zero horizontal divergence will exist. That "level of non-divergence" will be found at the height where the speed of the undisturbed current, v_x^* , is given by

$$v_x^* = c + \frac{2\Omega a \cos^3\varphi}{n^2} \tag{2}$$

In this equation c is the eastward speed of propagation of the wave, Ω is the angular velocity of the earth, a is the earth's radius, φ is the geographical latitude and n is the wave number per circumference of the earth. Numerical values of $2\Omega a \cos^3\varphi / n^2$ are given in Table II.

Table II

$2\Omega a \cos^3\varphi / n^2$ in m/sec

	Wave length in degrees longitude				
	180°	120°	60°	36°	18°
70°	9.3	4.1	1.0	0.4	0.1
60°	29.0	12.9	3.2	1.2	0.3
50°	61.6	27.4	6.8	2.5	0.6
40°	104.3	46.3	11.6	4.2	1.0
30°	150.7	67.0	16.8	6.0	1.5

Table II, in conjunction with equation (2), shows that in short waves, such as are found to accompany the individual traveling cyclone, the level of non-divergence lies at an elevation where the undisturbed current moves only a little faster than the wave. In long waves the air movement in the level of non-divergence goes much faster eastward than the wave itself, particularly so in low latitudes.

The level of non-divergence may be determined from sets of aerological maps by aid of Table II. Its height differs from case to case, but according to Charney ( ) and Cressman ( ) it averages around 600 mb both for long and short waves. Hence, with a given model of baroclinic westerlies, v_x^*, the speed of the undisturbed westerlies at the level of non-divergence, is approximately the same parameter for long and short waves. The speed of all such waves, which are superimposed on the same westerly current, therefore varies with wave length according to the formula

$$c = v_x^* - \frac{2\Omega a \cos^3\varphi}{n^2} \tag{3}$$

Short waves (n great) move almost with the speed of the air at the level of non-divergence. Long waves move slower eastwards than short ones, and may also retrograde. Table II applied to the case $c = 0$ gives us a survey of the wind at the level of non-divergence in stationary waves. In 70° latitude the west wind must be quite light for waves to be stationary, and the 180° wave length seems to be the most likely one for standing waves. Proceeding to lower latitudes we find that the 180° stationary wave requires stronger westerlies than are known ever to occur. Assuming that v_x^* would never be greater than 30 m/sec, we see that below 60° latitude the 180° stationary waves would never occur, below 50° also the 120° stationary waves become impossible, and so on. This dependence of the long wave pattern on geographical latitude usually leads to the establishment of only two or three standing waves per earth circumference near the pole and stationary patterns with higher wave numbers in lower latitudes. In the latitudes of pattern transitions complicated cases of wave interference occur.

The waves in the westerlies associated with the moving extra-tropical cyclones are by necessity of short wave lengths, say thirty degrees longitude. According to (3) and Table II such waves move with a speed, c , only slightly smaller than v_x^* , the speed of the westerlies at the level of non-divergence.

Figure 1 shows the position of the pressure minimum at sea level relative to that of the upper trough. The axis of minimum pressure of the closed low tilts towards the coldest side, which is usually to the west or northwest of the location of the surface center. The tropospheric part of the upper trough is also displaced ~~east~~westward with height but not as far per unit height as the subjacent center of low.

The friction layer of the closed vortex (up to 0.5 or 1 km height) has horizontal convergence. Above the influence of surface friction the eastern half of the vortex has horizontal convergence and the western half horizontal divergence of mass. This holds true also for the upper trough up to the level of non-divergence, beyond which divergence and convergence exchange positions. The tendency equation (1) applied to the schematic cyclone cross-section of Figure 1 gives an answer to the two questions: how can the cyclone move eastward as most middle latitude cyclones do? and how can it deepen despite the frictional convergence?

The eastward displacement of the cyclone is assured if the vertical integral of horizontal mass divergence in the tendency equation is determined as to sign by the atmosphere above the level of non-divergence. The deepening of the pressure minimum likewise depends on the influence from above the level of non-divergence. Due to the westward tilt of the axis of the cyclone a vertical air column located at the surface center will show horizontal convergence in its lower portion, where it passes through the forward part of the vortex, and horizontal divergence where it traverses the upper wave pattern east of the wave trough. Deepening of the surface center will only occur if that upper air divergence overcompensates the low level convergence.

In all parts of the cyclone the surface pressure tendency represents a small change of weight of the local vertical column resulting as the difference between accumulation and depletion of air each of which represent much greater weight changes. The natural adjustment of the pressure tendencies to the observed moderate values can be visualized as follows. In the low level vortex the front side convergence and the rear side divergence are more strongly developed the faster the vortex is forced to move. We have also seen from Table II that the level of non-divergence in the upper wave goes to higher elevation, and the divergence values above that level decrease, when the wave speed increases. Therefore, a supposed increase in speed without change in structure of the cyclone would lead to a weakening of the high level contribution and a strengthening of the low level contribution to the change in weight of air columns. That would be tantamount to a decrease in pressure tendencies. Quite analogously it can be shown that a supposed slowing down of the cyclone without change in its structure would lead to increasing surface pressure tendencies. From this can be concluded that the speed of a given cyclone is stable as long as its total three-dimensional structure remains the same. The pressure tendencies are then also stable although they are made up as small differences of large opposite contributions of horizontal divergence and convergence.

A real increase in the divergence effects of the upper wave would come from a lowering of the level of non-divergence and inherent increase of the ratio of the atmosphere in which the air current is supercritical. According to (3) that may take place through one of the following changes of parameters in the upper wave: (a) an increase of the speed, U_x , of the upper westerlies, (b) a decrease of angular wave length, $2\pi/m$ and (c) travel towards higher latitudes. In all these cases the compensating mass-divergence effects from low levels automatically pick up too as the speed of the cyclone increases. The occurrence of excessive values of barometric tendencies is thus automatically avoided. However, (a), (b) and (c) represent real dynamic processes which serve to increase pressure tendencies and cyclone speed.

The slowing down, which is normally observed in deep and extensive cyclones, is associated with the great depth of atmosphere moving in a closed cyclonic flow pattern. If that pattern has a subcritical eccentricity, which is the more frequent case, it will maintain mass-convergence in the eastern and mass-divergence in the western half. The influence of an upper wave pattern can then only barely overcompensate the divergence effects below, and the resulting pressure tendencies will be small. A final reversal of tendencies, and a retrograding of the cyclone, will result if the low level mass-divergence effects overcompensate those from the upper wave pattern.

The distribution of horizontal divergence also determines the vertical motion, which in the "smoothed" cyclone model always goes upward in the front half and downward in the rear.(as shown in Figure 1). That model feature agrees with the observed distribution of cloudiness and precipitation in the cyclones. Modern aerological analysis of the field of vertical motion carried on by the Meteorological Department of the New York University has corrobborated the same findings. A summary of the careful and extensive work in that field was published in 1948 by J. Miller ( ). From that report we also know that the vertical motion of extensive air masses usually is less than 3 cm/sec even at the level of non-divergence, where the maximum upward and downward values of momentum occur. Higher values of vertical motion up to 10 cm/sec should occur over narrow zones near fronts, while the occurrences of up- and downdrafts of several meters per second are restricted to small parts of individual convective clouds.

Typical patterns of temperature distribution in the extratropical cyclone can be seen from the sample cyclones described later in this article. The most frequent development of the temperature pattern in the lower tropospheric part of the vortex can be represented schematically by the maps and profiles in Figure 2. The incipient cyclone (Figure 2a) coincides with the apex of a warm tongue formed on a "front" across which the horizontal temperature gradient reaches a maximum. The frontal surface rises towards the cold side at an angle of inclination averaging around one in a hundred. It conserves its identity from day to day and moves along at a speed determinable from the winds through the kinematic boundary condition. The wave amplitude increases as the cyclone matures (Figure 2b) and the central pressure decreases. Next follows the "occlusion" process (Figure 2c and d) during which the warm tongue is lifted from the ground, first near the center, later also farther out. The occluded front formed at the junction of the two cold wedges tends to wrap around the cyclone center as part of a spiral, and the same shape is found for the warm tongue in all levels of closed cyclonic circulation. In the profile in Figure 2c, which is placed at a short distance south of the cyclone center, the occluded front is of warm front type, that is, the cold wedge behind the occlusion is less cold than the one in front. The same is the case in Figure 2d, but it can usually be assumed that farther south the occlusion is of cold front type. Where the transition from the one to the other occlusion model takes place the occluded front on the map must show a little gap. The lifting of a tongue of warm air relative to colder environment illustrated in Figure 2 can be assumed to furnish a great part of the increase in kinetic energy during the cyclone development from wave to vortex.

The tropopause is also shown in the profiles. It has a crest over the warm front surface and a trough over the cold front surface, and the amplitude of the tropopause oscillation increases with the growth of the cyclone. In Figure 2d the tropopause has a deep depression almost coinciding with the cyclone center, which is at that stage surrounded by air of cold origin up through the whole troposphere. Details of tropopause structure, like the frequent subdivision into multiple tropopauses have been left out in Figure 2.

Hatched areas in Figure 2 indicate the location of the main precipitation areas of the cyclone. The largest area is covered by the "warm front" rain, where the air from the warm tongue climbs the receding wedge of cold air and condenses much of its moisture. A more narrow zone of precipitation accompanies the "cold front" where some air from the lower part of the warm tongue is lifted by the advancing cold wedge. Higher portions of the warm tongue move faster than the cold front wedge and are not lifted by it. The described upward motion of the warm air next to the frontal surfaces should be visualized as being superimposed on the general pattern of vertical motion, upward in the front half and downward in the rear half of the cyclone (Figure 1). The upward motion of that general nature is sometimes sufficient to cause rain also where it is not called for as a consequence of upgliding on frontal surfaces. Some extensive "warm sector rains" and also the rain in the front half of a cold trough or a cold vortex are probably to be explained by the general upward motion shown in Figure 1.

To complete the precipitation picture of the cyclone the "air mass precipitation" should also be added: the drizzle in the warm moist parts condensing from low cloud formed by the cooling of the warm air over cold surface (mainly ocean surfaces), and the convective showery precipitation formed through the heating from the ground, or through lifting of convectively unstable air at fronts.

While the thermal pattern of the cyclone near the ground is the result mainly of horizontal advection and non-adiabatic gain or loss of heat exchanged with the ground, the pattern in the free atmosphere is also influenced by the slow but systematic vertical displacements of the air shown in Figure 1 and by the heat transfers of penetrative convection. However, the dominating process for the shaping of the upper tropospheric temperature field is still the horizontal advection. The development of the thermal pattern of the waves in the upper westerlies follows roughly the advective scheme in Figure 3. A warm tongue forms in the part of the wave with advection from the south, and a cold tongue in the part with advection from the north. In this early stage of the wave the pressure crests and troughs must tilt westward, as shown for the pressure trough in Figure 1. In the further development both warm and cold tongues grow in amplitude and move forward relative to the pressure wave, because the eastward motion of the air exceeds that of the wave. If the wave motion were entirely horizontal a thermal pattern of permanent structure relative to the moving wave would ~~be~~ prevail if ~~reached when~~ the isotherms ~~have adopted to the shape of~~ coincided with the relative streamlines. For the idealized case of v_x = constant in each level of the sinusoidal wave pattern the ratio of the amplitude, A_R , of the relative streamline to that of the streamline, A_S , would be

$$\frac{A_R}{A_S} = \frac{v_x}{v_x - C} \tag{4}$$

Hence, close to the level where v_x is equal to the wave speed, C , the relative streamlines, ~~and with them the advectively transported isotherms,~~ would acquire a much greater amplitude than that of the streamline, and the amplitude of the advectively transported isotherms would increase even more. The ratio A_R/A_S will decrease from that level upward to the tropopause, where v_x has its maximum.

Under the influence of the upward motion ahead of the pressure trough, and downward motion behind it, the ratio in (4) would be reduced, as shown by J. Miller ( ). Synoptic experience shows that A_R/A_S stays positive in all tropospheric levels where $v_x > C$ also under the joint influence of vertical motion and horizontal advection. In other words, after some time of thermal transformation in the style of Figure 3, the waves in the upper troposphere tend to become thermally symmetric with warm tongues coinciding with their pressure crests and cold tongues with their pressure troughs.

With the reversal of the meridional temperature gradient from troposphere to stratosphere the advective effects on temperature in the upper waves are reversed too. Hence, the stratospheric pressure crests are cold and the pressure troughs warm. Indirectly follows then also that the wave pattern of pressure crests and troughs rapidly loses amplitude with height in the stratosphere. In the stably stratified stratosphere the local warming and cooling through vertical motion are stronger than in the troposphere, and are quite often stronger than the temperature change by advection.

b. Vorticity analysis of the extratropical cyclone. An analysis of the vorticity distribution and the history of vorticity change of individual parcels in the vortex and wave will reveal more of the dynamics of the cyclone. Vorticity about the vertical axis, ζ , may be identified on the horizontal streamline maps as a particle rotation about a vertical axis partly due to curvature, v/r_s , where r_s is the radius of curvature of the streamline, and partly due to shear, $-\partial v/\partial n$,

$$\zeta = \pm\frac{v}{r_s} - \frac{\partial v}{\partial n} = \frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} . \tag{5}$$

The rules for the use of sign can always be decided by referring to the cartesian component form of vorticity added as an alternate expression in (5). The convention used here is to let the positive direction of the coordinate n point to the left of the wind, to consider v and r_s always positive, and to use the positive sign in front of v/r_s for cyclonic and negative for anticyclonic curvature of the streamlines.

The vorticity change of the individual traveling particle is given by the equation*) **)

*) Hesselberg, Th. and Friedmann, A. (13), Rossby, C.-G. (26,27), Haurwitz, B. (11), Holmboe, J. (16).

$$\frac{d\zeta}{dt} = \frac{\partial p}{\partial x}\frac{\partial \alpha}{\partial y} - \frac{\partial p}{\partial y}\frac{\partial \alpha}{\partial x} - (\zeta + 2\Omega \sin\varphi)\, div_H \mathbf{V} - \frac{2\Omega\cos\varphi}{a} v_y + \frac{\partial v_z}{\partial y}\left(2\Omega\cos\varphi + \frac{\partial v_x}{\partial z}\right) - \frac{\partial v_z}{\partial x}\frac{\partial v_y}{\partial z} . \tag{6}$$

The first two terms to the right represent in component form the effect of isobaric-isosteric solenoids on the change of vertical vorticity. Those terms are always found to be insignificant compared to the following ones. The divergence term shows how horizontal expansion (divergence) creates negative (anticyclonic) vorticity, and horizontal contraction creates positive (cyclonic) vorticity. The next term shows the effect

**) Where friction is neglected.

on relative vorticity, ζ, of the displacement of the air either towards the polar regions, where the vertical component of earth vorticity, $2\Omega\sin\varphi$, is great, or towards the equator, where $2\Omega\sin\varphi$ is zero. In absence of the other factors the case of poleward movement would entail decrease of relative cyclonic vorticity or increase of relative anticyclonic vorticity, and the movement away from the pole increase of relative cyclonic or decrease of relative anticyclonic vorticity. The two last terms on the right hand side describe the influence of the vertical motion in changing the vorticity about the vertical. The term in $\frac{\partial v_z}{\partial y}$ represents in part the fact that the infinitesimal disk of air, whose rotation decides the value of ζ, arrives to the horizontal position from earlier positions with meridional tilt. In terms of relative vorticity change that is equivalent to a change in latitude in addition to that by horizontal meridional advection, as can be seen from the analogy of the terms $2\Omega\cos\varphi\frac{\partial v_z}{\partial y}$ and $-2\Omega\cos\varphi\frac{v_y}{a}$. Furthermore, the term in $\frac{\partial v_z}{\partial y}$ represents the effect of rotating the air in the yz-plane so that the vorticity about the y-axis, $\frac{\partial v_x}{\partial z}$, gets a vertical component at the rate $\frac{\partial v_z}{\partial y}\frac{\partial v_x}{\partial z}$ per unit time. Analogously, the last term in (6) represents the rate of change of vorticity about the z-axis resulting from a rotation of the air in the xz plane. Usually the terms in $\frac{\partial v_z}{\partial y}$ and $\frac{\partial v_z}{\partial x}$ are considered to be insignificant in relation to the divergence term and meridional advection term, but possible exceptions will be mentioned below.

In the frontal wave of the lower troposphere the cold air enters the cyclone along the warm front (Figure 4). Next to the front it has initial cyclonic shear which has been acquired during the period of frontogenesis (see p.). The increase of the cyclonic vorticity in the cold air on its way toward the wave apex is due to the horizontal convergence, which extends all over the front half of the cyclone (see Figure 1). That creation of relative cyclonic vorticity is somewhat reduced by the effect of the poleward component of air travel. Behind the wave apex the air returns southward whereby its relative cyclonic vorticity should increase. At the same time, however, the air enters the region of horizontal divergence, which has the opposite effect upon vorticity change. The result is that the air which had acquired maximum cyclonic shear along the warm front maintains cyclonic vorticity also after passing the wave apex, but with a simultaneous shift from shear to curvature vorticity. The cold air passing at greater distance from the center changes from moderate cyclonic to anticyclonic vorticity. During the growth of the cyclone more and more of the cold air is able to maintain its cyclonic vorticity after passing the wave apex.

The warm air in the frontal wave enters the cyclone from the southwest. Its speed in lower levels just barely exceeds that of the cyclone in its eastward motion. Upon arrival at the warm front the warm air climbs the receding cold wedge. Again, one branch of anticyclonic and another of cyclonic vorticity may be discerned. Farthest away from the center, where the horizontal convergence is moderate or non-existent, the warm air gains anticyclonic relative vorticity through the poleward component of movement. Closer to the center, where the horizontal convergence is stronger, the warm air acquires cyclonic relative vorticity despite its displacement polewards. This latter development, which does not start until the cyclone is over the nascent stage, gains in volume with the growth of the cyclone.

In the westerly wave of the upper troposphere, as represented in Figure 1, the air enters the cyclone from the northwest and leaves it eastward bound across the wave crest ahead of the surface cyclone. As long as the upper wave does not degenerate, the relative vorticity changes sign at the longitude of the inflexion points, thus changing from anticyclonic to cyclonic relative vorticity in the middle of the zone of convergence, and from cyclonic to anticyclonic in the middle of the zone of divergence. This divergence effect (second term to the right in (6)) overcompensates the effect of changing latitude in the ordinary short-wave and small-amplitude sinusoidal disturbance of the upper troposphere and lower stratosphere.

The factor $(\zeta + 2\Omega \sin\varphi)$ in the divergence term of (6) is equal to the absolute vorticity, ζ_a, of the air relative to a non-rotating coordinate system. When ζ is positive (cyclonic) ζ_a is big and the individual vorticity change with time is quite sensitive to horizontal convergence or divergence. If the air is subject to a sustained process of horizontal convergence its cyclonic vorticity will increase without any theoretical upper limit. The cyclonic bends of an upper sinusoidal westerly are therefore frequently seen to become more strongly curved than the anticyclonic bends. When ζ is strongly negative (anticyclonic) the absolute vorticity may go to zero or even become negative. This happens almost exclusively in the upper troposphere and lower stratosphere where the wind velocities are very strong. On such wave crests where ζ_a reaches values close to zero, the vorticity change is only feebly influenced by horizontal divergence, 6 and obeys only the first right hand term in (5). That would be equivalent to motion under constant absolute vorticity $\zeta_a \simeq 0$ or $\zeta \simeq -2\Omega \sin\varphi$. If a wave crest in the upper atmosphere has developed to that extreme stage, the particles overtaking the crest would maintain their anticyclonic vorticity (in the form of curvature and/or shear) for a long period thereafter. Figure 5 illustrates schematically that case. From an initial flow pattern of sinusoidal westerlies (streamline 1) a "meandering" westerly current develops through the growth of the wave crest and the deepening of the next downwind wave trough (streamlines 2 and 3). This development towards meandering flow is not dependent on the absolute vorticity actually having reached zero. Also with absolute vorticities still positive, but numerically small, the vorticity change begins to react sluggishly on horizontal divergence with the result that the sinusoidal perturbation of the westerlies begins to degenerate. The meandering development may also start from an initially straight current with anticyclonic shear close to the value $-2\Omega \sin\varphi$. Any small wave impulse may then develop into meandering wave patterns.

It is obvious that the meandering phenomenon, once started in regions of excessive anticyclonic vorticity in the upper atmosphere, will be of profound influence also on the total cyclone picture down to the ground. The deepening of the upper wave trough is associated with the deepening of the cyclonic vortex underneath, and usually also entails a southward component added to the normal eastward displacement of the cyclone. In all cases of such deepening the initial upper disturbance must start on the wave crest to the west of the cyclone.

Always?

Above the level of non-divergence the divergence term and the meridional advection term in (6) are of the same phase, so that the additional terms in $\frac{\partial v_z}{\partial y}$ and $\frac{\partial v_z}{\partial x}$ are not likely to affect the general pattern of $\frac{d\zeta}{dt}$ very much. The two levels where their influence may be expected to count are: firstly, close to the level of non-divergence, secondly at the localities where $\zeta - 2\Omega \sin\varphi$ is near zero. In both of those cases the competition with the divergence term is almost eliminated. Furthermore, v_z and its horizontal derivatives reach their maximum in the upper troposphere (about two km above the level of non-divergence where $|\varrho v_z|$ has its maximum).

The most compelling reason for admitting a perceptible influence of the terms in $\frac{\partial v_z}{\partial y}$ and $\frac{\partial v_z}{\partial x}$ on the variations of ζ lies in the fact that neither the divergence term, nor the meridional advection term, nor their sum, can account through equation (6) for the occurrence of negative absolute vorticity. Analysis of observational data do show areas of negative absolute vorticity on pronounced upper wave crests and/or south of pronounced "jet streams" (see figure 7). A vertical motion effect of the right sign to explain the growth of $-\zeta$ beyond $2\Omega \sin\varphi$ would be found north of the maximum of upward velocity in the cyclone ($\frac{\partial v_z}{\partial y} < 0$, $\frac{\partial v_z}{\partial z} > 0$). The result in terms of a big anticyclonic vorticity, and occasionally a negative absolute vorticity, can then be expected to accrue on the upper wave crest to the east of the cyclone.

The above reasoning about the vertical motion terms in equation (6) has been developed by L. Sherman and will appear under his authorship.

c. Inertial motion in isentropic surfaces. In slow-moving long waves of the upper westerlies the streamlines relative to the waves almost coincide with the streamlines relative to earth and the isotherms will be moved advectively so as to coincide more or less with the isobars. Around the inflexion points of such long waves we find the best approximation to the relatively simple conditions of straight baroclinic flow. Frontogenesis and frontal cyclogenesis are frequent under the straight southwesterly currents of long waves, and for a study of these phenomena we will here consider the theory of adiabatic, inertial motion in tilting, stationary isentropic surfaces. Adiabatic (or pseudo-adiabatic) changes of state of particles in the free atmosphere can justifiably be assumed, because non-adiabatic temperature changes are so slow and uniformly distributed that they do not much affect a swift phenomenon like cyclogenesis.

The following simplified analysis has been inspired by the theoretical studies on "dynamic instability", which go back to the classical paper of H. Helmholtz ( ) "Über atmosphärische Bewegungen" in 1888. Several recent contributions by theoretical meteorologists to the same field have been included in the list of literature references (Nos.). The synoptic applications of equation (11) in this article to the problems of frontogenesis and cyclogenesis have to my knowledge not been attempted before.

The adiabatic movement of a particle parallel to an isentropic surface (sloping or horizontal) is not opposed by buoyancy forces and is left to the stable or unstable control of the horizontal pressure gradient and the Coriolis force. The same is true for particles of entire isentropic sheets moving in unison, when they are far enough from the ground to be independent of boundary effects. For the purpose of this article we will consider the environment field of pressure and potential temperature constant while the sample particle (or sample isentropic sheet) moves isentropically through the field. We will consider part of an isentropic surface of sufficiently small extent to be treated as a sloping plane (Figure 6), on which the horizontal direction will be called the x-direction (due eastward) and the direction of steepest slope (due northward) will be called the η-direction. The η-axis is supposed to form an angle with the horizontal y-axis of the order of one in a hundred or less. The undisturbed air motion is supposed to be zonal, geostrophic, and horizontal, which allows the isentropic surface to stay fixed in space. Furthermore the fundamental motion is constant along each streamline, $\partial v_x/\partial x = 0$, and represents a steady state, $\partial v_x/\partial t = 0$. Later application to the long wave, where the quasi-straight flow is not exactly zonal, will be done without any strict mathematical treatment. The disturbed motion of the sample particle is supposed to be contained in the isentropic surface and to have an initial up-slope component, v_η, superimposed on the general horizontal motion characteristic of the environment.

The x-component of acceleration of the disturbed particle amounts to

$$\frac{dv_x}{dt} = 2\Omega_z v_y \cong 2\Omega_z v_\eta , \qquad (7)$$

and makes the particle speed up in the positive x-direction while it climbs the isentropic slope. The wind of the environment, v_g, which is geostrophic and directed along x in the whole field, changes in value along the path of climb. An observer following the disturbed particle would see the environment geostrophic wind relative to earth change by

$$\frac{dv_g}{dt} = v_\eta \frac{\partial v_g}{\partial \eta} . \qquad (8)$$

The χ-component of the speed of the disturbed particle was assumed to be equal to the geostrophic wind, v_g, at the initial time. Depending on whether $dv_x/dt > dv_g/dt$ or $dv_x/dt < dv_g/dt$ the disturbed particle will go faster, or slower, eastwards than its new environment after a time differential of climbing. In the first case the y-component of acceleration of the disturbed particle, $dv_y/dt = -2\Omega_z(v_x - v_g)$, will be directed down the isentropic slope opposite to the initial disturbance. A stable inertia type of oscillation will then result. In the other case the y-component of acceleration will point in the same direction as the initial disturbance velocity v_y, so that an exponential growth of the disturbance will follow. The instability case is thus $dv_x/dt < dv_g/dt$. In order to make the instability criterion applicable also for the downward directed disturbance, it should be written $\left|\frac{dv_x}{dt}\right| < \left|\frac{dv_g}{dt}\right|$.

Now, provided that the substitution of v_y for v_g in (7) is justified by a sufficiently small inclination of the isentropic surface, the instability criterion derived from (7) and (8) takes the simple form

$$\frac{\partial v_g}{\partial y} > 2\Omega_z . \tag{9}$$

The observed increase of westerly geostrophic wind from the lower to the higher portion of an isentropic surface sometimes satisfies the above instability criterion, as will be shown later.

If we drop the initial conditions of $\partial v_x/\partial x = 0$ and $\partial v_x/\partial t = 0$, no exact treatment can be offered, because then the fundamental current is not a steady state one. Even so, the following qualitative reasoning will point out in which sense the criterion of inertial instability would change by the introduction of $\partial v_x/\partial x \lessgtr 0$ and $\partial v_x/\partial t \lessgtr 0$.

In Figure 6 an element of the isentropic surface is shown in

$x\eta$-coordinates. Isobars on that surface are parallel to the x-direction, while the distribution of the speed of the geostrophic wind, v_g, is shown by slanting scalar curves. v_g has thus a gradient in x-direction in addition to the much stronger gradient in η-direction. The disturbed path of a sample particle along the isentropic surface is supposed to go from A to B during the time differential. The x-component of the acceleration (parallel to the isobars) of the sample particle is again in first approximation $\frac{dv_x}{dt} = 2\Omega_z v_\eta$, while the change of geostrophic wind encountered along the path is

$$\frac{dv_g}{dt} = v_\eta \frac{\partial v_g}{\partial \eta} + v_x \frac{\partial v_g}{\partial x} + \frac{\partial v_g}{\partial t}. \tag{10}$$

The acceleration of the particle in η-direction, is supposed to be zero at the beginning point of the trajectory A. The acceleration in η-direction will be zero also at the end of the trajectory, B, if $\frac{dv_x}{dt} = \frac{dv_g}{dt}$, or

$$2\Omega_z v_\eta = v_\eta \frac{\partial v_g}{\partial \eta} + v_x \frac{\partial v_g}{\partial x} + \frac{\partial v_g}{\partial t},$$

that is:

$$v_\eta = \frac{v_x \frac{\partial v_g}{\partial x} + \frac{\partial v_g}{\partial t}}{2\Omega_z - \frac{\partial v_g}{\partial \eta}} \tag{11}$$

Specializing now for the condition $2\Omega_z - \frac{\partial v_g}{\partial \eta} > 0$ and for $v_x \frac{\partial v_g}{\partial x} + \frac{\partial v_g}{\partial t} > 0$, we find that the particle given an initial speed component, v_η, greater than the value found in (11) will have an acceleration component $\frac{dv_\eta}{dt}$ opposite to v_η. Given a smaller positive initial v_η, or a negative initial v_η, the particle would accelerate towards the value given in (11) for v_η. That value of v_η therefore represents the η-component of a stable up-gliding motion in which all the particles of the isentropic surface may join. The η-component of the stable up-gliding motion approaches infinity when $2\Omega_z - \frac{\partial v_g}{\partial \eta}$ goes to zero. In other words, in the case of inertial indifference any finite initial v_η would increase exponentially.

Quite analogous reasoning with the case $2\Omega_z - \frac{\partial v_g}{\partial \eta} > 0$ and $v_x \frac{\partial v_g}{\partial x} + \frac{\partial v_g}{\partial t} < 0$ reveals the existence of stable down-gliding motion whose η -component is also given by (11).

The difference between the cases $v_x \frac{\partial v_g}{\partial x} + \frac{\partial v_g}{\partial t} = 0$ and $\gtrless 0$ is thus the following: In the former case the departures from the geostrophic wind remain of a stable oscillatory nature until the anticyclonic isentropic shear reaches the critical value of $-2\Omega_z$. In the latter cases there is a sustained stable departure from the geostrophic wind, represented by v_η , which has finite values also when $2\Omega_z - \frac{\partial v_g}{\partial \eta} > 0$. In addition there may be inertial perturbations superimposed on the current representing the vector sum of geostrophic motion, v_g , and isentropic motion, v_η ; and such perturbations will be stable as long as $2\Omega_z - \frac{\partial v_g}{\partial \eta} > 0$.

The demonstration of the occurrence of anticyclonic shear in the upper atmosphere, which approahces or even surpasses the critical limit of dynamic instability, is due to recent research work at the University of Chicago. The first profile of the westerlies showing such conditions was analyzed by E. Palmén (23) in 1948. Since then many profiles across straight baroclinic westerlies with a stationary polar front have been studied, leading to the crystallization of a model of the straight westerlies reprinted here as Figure 7. The left part of that diagram is an is~~entrope~~otherm-isovel profile based on an averaging of twelve eastern North American meridional profiles during December 1946, and published by E. Palmén and C. W. Newton (24). In the right hand part of Figure 7 we have added a diagram of the computed quantity $2\Omega_z - \frac{\partial v_g}{\partial \eta}$, with η interpreted as the curv~~i~~e-linear isentropic coordinate (positive direction northwards). In most of the field $2\Omega_z$ is greater than $\frac{\partial v_g}{\partial \eta}$, indicating inertial stability; but in a narrow zone south of the maximum upper westerlies $2\Omega_z - \frac{\partial v_g}{\partial \eta}$ is negative, indicating inertial instability. Furthermore, in the frontal zone below 600 mb, where $\frac{\partial v_g}{\partial y}$ has been measured along saturation-isentropes, the values of $2\Omega_z - \partial v_g/\partial y$ indicate only a slight amount of inertial stability. We will first focus our attention on that part of the profile.

The small positive values of $2\Omega_z - \partial v_g/\partial y$ (or in some individual cases negative ones) are located in a narrow frontal zone, while in the adjacent parts of the warm and cold air masses $2\Omega_z - \partial v_g/\partial y$ is positive and far from zero. Since in (11) the component of stable up- or down-gliding is inversely proportional to $2\Omega_z - \partial v_g/\partial y$, it follows that the air in the narrow frontal zone has a much greater possibility for isentropic up- or down-displacements than the air masses on either side.

The quantity $v_x \partial v_y/\partial x + \partial v_y/\partial t$, representing the numerator in the expression for v_y in (11) cannot be judged from the data of one profile alone. It will be great and positive firstly where the isobars of the horizontal pressure distribution converge, and, secondly, where the gradient wind increases locally with time. The first condition is fulfilled for instance along the axis of kinematic dilatation extending eastward from a col of the pressure field. That synoptic situation is known to be frequently associated with frontogenesis and subsequent maintenance of a sharp front. The second condition, local increase of gradient wind parallel to the frontal zone, frequently occurs during frontogenesis, but the cause of such increase of gradient wind is not necessarily attributable to the front mechanism.

Whenever $v_x \partial v_y/\partial x + \partial v_y/\partial t$ has the same sign in each of the two air masses v_y will also have the same sign in the whole field; but its maximum numerical values, as far as the lower troposphere is concerned, will be found in the frontal zone where $2\Omega_z - \partial v_y/\partial y$ is at a minimum.

As shown in Figure 7, the warm air over the lower and medium portion of the polar front surface has anticyclonic isentropic shear, increasing to great values in the upper troposphere, whereas the air above the upper part of the frontal surface has cyclonic shear, likewise increasing to high values in the upper troposphere. The dividing line between anticyclonic and cyclonic shear runs almost vertically through the maximum of west wind velocity, which in the average condition represented by Figure 7 is located above the place where the frontal surface intersects the 500 mb level. Isentropic up- or down-gliding as defined by equation (11) will reach bigger values south of the velocity maximum than north of it. It is likely that this difference in v_η values north and south of the velocity maximum does give rise to important horizontal divergence effects because the η -component represents a non-geostrophic part of the total wind. The v_η -divergence effect in the jet stream region should work out as shown schematically in Figure 8. Where there is "confluence" of the winds into the western beginning of a "jet stream", equation (11) indicates a superimposed isentropic up-gliding, $v_\eta > 0$, and "isentropic convergence", $\frac{\partial v_\eta}{\partial \eta} < 0$ north of the line of maximum $|v_\eta|$. Where the wind velocity decreases along the streamlines in the "delta" of a jet stream, equation (11) indicates isentropic down-gliding, $v_\eta < 0$, and isentropic divergence, $\frac{\partial v_\eta}{\partial \eta} > 0$, north of the line of maximum $|v_\eta|$. In the stratosphere, because of the opposite tilt of isentropic surfaces, the terms down-gliding and up-gliding must be interchanged, but the statement about the isentropic divergence remains identical for stratosphere and troposphere. At the line of maximum $|v_\eta|$ values the isentropic divergence $\frac{\partial v_\eta}{\partial \eta}$ changes sign, as shown by the shading in Figure 8.

In figuring out the effect of the isentropic divergence in changing the pressure field we may think of the distribution of $\frac{\partial v_\eta}{\partial \eta}$ as representing in the first approximation a field of $\frac{\partial v_y}{\partial y}$, where v_y is the non-geostrophic y -component of motion. Assuming that the distribution of $\frac{\partial v_y}{\partial y}$ can also qualitatively be represented by the shaded and unshaded areas in Figure 8, we have in that diagram an outline of the contribution of isentropic divergence to the total horizontal divergence. The isentropic divergence, acting in the same sense through the stratosphere and the upper half of the troposphere, may be an important effect to consider together with the divergence effects represented in Figure 1. A cyclonic storm traveling along the jet stream zone would come under the influence of superimposed upper mass divergence from the time when it passes the place of greatest constriction of the upper streamlines. A complete theoretical treatment of this case, which calls for a combination of the divergence effects of Figure 1 and Figure 8, is not available; but there seems to be considerable empirical evidence for strong cyclone deepening under the described circumstances. Such synoptic evidence has mainly been gathered by R. Scherhag ( ). Scherhag points to V. H. Ryd ( ) as the originator of the idea that mass divergence of importance for cyclone deepening should occur in upper delta patterns. Ryd's theoretical contributions appeared in 1923 and 1927 when there were as yet no upper air maps.

Returning to Figure 7 we see that complete inertial instability, $\frac{\partial v_q}{\partial q} > 2\Omega_z$, may at times extend from 150 mb down towards the 500 mb surface. It may also extend over something like a thousand kilometers length of current, but the width of the zone of such unstable shear is hardly more than three hundred kilometers at any one point. Inside that volume of current the geostrophic wind, with its superimposed isentropic up- or down-gliding component, does not represent a stable flow. However, with stable neighboring flow on either side, no very large unstable deviations from geostrophic flow will be able to develop. The most likely system of perturbations in the unstable part of the current will be cellular helical circulations, as indicated in Figure 7. Such circulations would serve the purpose of exchanging momentum across the zone of unstable shear and thus lessen that shear. The height of each cell would have to be small, probably less than one km, so that the solenoid field set up by the cellular circulation should not grow strong enough to reverse the initial circulation. An indirect indication of the existence of the helical cellular circulations is seen in the observed "multiple tropopauses", each one probably representing a cell wall between superjacent circulation rolls. According to Palmén ( ) these multiple tropopauses are quasi-isentropic as could be expected if they are formed as circulation-cell boundaries.

2. SYNOPTIC EXAMPLE OF AN EXTRATROPICAL CYCLONE.

The weather situation over North America during November 7-10, 1948, has been selected to illustrate the principles of this article. During the period a large occluded cyclone over the Hudson Bay region can serve as a model of the most frequent structure of old cyclones, while over the central United States the atmosphere displays all the successive stages of frontogenesis and the early life history of a growing frontal wave cyclone. Our description will begin with the evolution of the long-wave background pattern of the upper layers represented by a set of 300 mb maps, then the advective frontogenesis in the lower troposphere will be illustrated by a sequence of ground-level and 850 mb maps as well as selected profiles, and finally the three-dimensional structure of the frontal wave cyclone will be shown by a synoptic set of maps from the ground to 300 mb.

2a. SYNOPTIC EVOLUTION OF THE UPPER LAYERS.

The six 300 mb maps at 12h intervals in Figure 9 all show the semipermanent Hudson Bay cyclone. Through the whole troposphere that cyclone is a cold-core vortex and therefore shows up as a deep center on the 300 mb maps. Equally permanent is the crest of high pressure extending northwards from a warm anticyclone over the eastern North Pacific. Both the Hudson Bay low and the east-Pacific high are typical features of the general circulation only somewhat over average in strength during November 7-10. The westerly current meandering through between them is quite strong over a narrow zone, while the pressure gradients in the high and the low are quite weak. The trough located over the western United States on November 7 moves slowly to the central states and deepens gradually from November 7 to November 9. This upper air process plays an important role in the formation of the frontal cyclone which takes place under the pre-trough southwesterly current (without producing any separate low pressure center at 300 mb).

The deepening of the upper trough may be caused in two ways (see equation (1)): either through a sinking component of motion at the 300 mb level, or through horizontal mass divergence in the column above 300 mb. In the former case the temperature in the trough at 300 mb ought to be rising with time. This is not borne out by the observations during November 7-9, so that we are left with the horizontal mass-divergence as the probable cause of the deepening of the trough. The mass-divergence must be operated through the feeding of air into the trough with such a high velocity that the Coriolis force and centrifugal force overcompensate the initial pressure gradient. The mechanism for producing such a strong jet in the northerly current behind the trough must be sought on the anticyclonic bend to the west.

The maximum curvature of the 28400 foot contour of the 300 mb surface is represented on the November 7th map by an arc of circle with radius r_i. At the same place in western Canada the maximum possible curvature of a steady state anticyclonic current, flowing under the influence of the observed pressure gradient force, is represented by another arc of circle with radius r_{min} *). The curvature analysis

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*) The value of r_{min} is obtained from the equation of anticyclonic circular motion

$$-\frac{v^2}{r} = -2\Omega_z v - \frac{\partial \phi}{\partial r} = -2\Omega_z (v - v_g), \qquad (12)$$

in which Φ stands for the geopotential in the pressure topography, r and v are positive, $\partial\phi/\partial r$ is the outward directed pressure gradient, $-2\Omega_z v$ is the inward directed Coriolis force, and $-v^2/r$ is the centripetal acceleration. When equation (12) is applied to a selected point of the map, Ω_z and $\partial\phi/\partial r$ are constants. Solving (12) for r

$$r = \frac{v^2}{2\Omega_z v + \partial\phi/\partial r} = \frac{v^2}{2\Omega_z (v - v_g)} \tag{13}$$

and seeking the value of v for which r is a minimum, gives

$$v = \frac{-\frac{\partial\phi}{\partial r}}{\Omega_z} = 2 v_g . \tag{14}$$

The corresponding values of r_{min} and v are thus

$$r_{min} = -\frac{\frac{\partial\phi}{\partial r}}{\Omega_z^2} = \frac{2 v_g}{\Omega_z}, \quad v = 2 v_g . \tag{15}$$

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on the 300 mb anticyclonic bend over Western Canada on November 7 and the following days shows that instances of $r_i < r_{min}$ are quite frequent, in other words that with the given pressure gradient and contour curvature the paths of particles often cannot have as small a radius of curvature as that of the isobaric contours. If that applies to a quasi-stationary pressure ridge like the orographic one over the Canadian Rockies, where the radius, r_s, of streamline curvature is equal to the radius, r, of path curvature, the air would have to cross the isobars towards low pressure while making the anticyclonic turn. This must imply a forward acceleration of the particle leading up to maximum speed at the end of the anticyclonic sweep. If such fast moving air is fed directly into a pressure trough ahead, whose pressure gradients were adapted to smaller wind speeds, a deepening of the trough should follow. The deepening of the large pressure trough over the western and central U.S. during November 7-9, 1948, should probably be interpreted that way.

The flow around the quasi-stationary anticyclonic bend over western Canada cannot be a steady state one although the major features of that part of the map do remain unchanged. The 300 mb maps show how the one moving wave perturbation after the other appear on top of the large stationary crest of high pressure in the west. The first of these traveled about 1500 km during twelve hours (35 m/sec) and is found on the second map with its wave crest in the northerly current at the Canadian-U.S. border. The 300 mb contour curvature at that time and place defines an r_i which is much smaller than r_{min}.

The radii of curvature of streamlines, r_s, and of air trajectories, r, in a moving sinusoidal wave are tied to each other by the formula

$$r_s = r \frac{v - c}{v}, \tag{16}$$

where c is the speed of the wave. No measured wind velocities are available at 300 mb in western Canada during the days under consideration. Theoretical estimates of v must lie between v_g and $2v_g$, most likely closer to the lower than the upper limit, as will be shown later. Assuming tentatively for the moving pressure crest at the U.S.-Canadian border on November 8, 0300Z, $v = 1.1\, v_g = 49$ m/sec, we would have from (13)

$$r = \frac{49^2}{1.08 \cdot 10^{-4}\,(49 - 44.5)} = 4900 \text{ km},$$

and would arrive at the following estimate of r_s :

$$r_s = 49 \cdot 10^5 \, \frac{49 - 35}{49} \, m = 1400 \text{ km}.$$

which is much longer than the measured radius of contours, $r_i = 440$ km. These estimates and measurements are of course subject to great errors, but even so the conclusion seems to be that also on the moving pressure crests the streamlines will fail to adapt to the strong curvature of the isobars. It then also follows that the moving pressure crests in the 300 mb level are preceded by a velocity maximum. When the air from that velocity maximum enters the slow moving low pressure trough, a pulse of deepening by centrifugal action would result. The rapidly moving upper wave cannot be seen to continue its propagation on the front side of the deep slow moving trough. Hence all its wave energy must have been absorbed in the big trough.

With the above estimate of $v = 49$ m/sec and $r_s = 1400$ km the anticyclonic vorticity due to curvature $-v/r_s$ amounts to $-3{,}5.10^{-5}$ sec^{-1}. This is numerically much less than $2\Omega \sin 50° = 1{,}1.10^{-4}$ sec^{-1}, so it does not seem likely that the complete anticyclonic vorticity $-\frac{v}{r_s} - \frac{\partial v}{\partial n}$ reaches the critical value of $-2\Omega_z$ anywhere in the rapid wave at 300 mb. The described manifestation of instability through cross-isobaric flow on the anticyclonic bend thus takes place independently of the fulfillment of the criterion $-\frac{v}{r_s} - \frac{\partial v}{\partial n} + 2\Omega_z < 0$.

On November 8, 1500Z, when the most unstable part of the anticyclonic flow was found far north (again marked by $r_i < r_{min}$), a growing crest and a downwind deepening trough formed simultaneously. When that perturbation caught up with the slow moving trough ahead another deepening occurred (see November 9, 1500Z), this time in the north-central U.S. while the southern end of the trough at that time was losing depth.

The described fast moving unstable waves on the 300 mb maps are, of course, at times connected with disturbances in the lower atmosphere. The first of the upper waves was formed on November 7, 1500Z, as an occluded front was approaching from the west; and it is likely that the excessive anticyclonic curvature resulted from a superposition of the upper wave crest associated with the occluded cyclone upon the semi-permanent anticyclonic bend which is tied orographically to the northern Rocky Mountains. Once formed, the unstable upper wave separates from the frontal disturbance by virtue of its superior speed (35 m/sec). The second unstable upper wave had no clear connection with any frontal disturbance.

During the selected period the flow of air east of the big slow moving trough turned gradually from WSW towards SSW while increasing a little in strength. On November 8, 1500Z, after the cold trough of the Hudson Bay cyclone had moved off to the northeast, the upper current over the eastern half of the U.S. and Canada became almost straight. On November 9, 0300Z, when the growing frontal cyclone (marked by an asterisk on the 300 mb maps) began to exert influence high up, the upper current became slightly S-shaped. The new-formed upper wave moved along with the cyclone center below at a speed of only 9 m/sec. The best estimate of the wind speed on the anticyclonic bend is probably 80 m/sec (see below) and hence $r_s = r \frac{80-9}{80} = 0.9\, r$. Even with $r = r_{min}$ = 1950 km r_s would be 0.9 . 1950 km = 1860 km, which is greater than the measured r_i = 1350 km. The streamlines will consequently not be able to adapt to pressure contours around the anticyclonic bend.

The above tentative assumption of $r = r_{min}$ is really predicated on the further assumption that the wind maintains a speed of $2v_g$ around the anticyclonic bend. We can in this case show convincingly that the wind does not reach such a speed and therefore also that the air trajectory must have a radius of curvature considerably longer than r_{min} .

The geostrophic wind in the strongest part of the straight southwesterly current on November 8, 1500Z, amounted to about 70 m/sec, and on November 9, 0300Z, a geostrophic measurement in the Great Lakes region gives nearly the same value. Even at the geostrophic speed of 70 m/sec, which gives a speed of 70-9 = 61 m/sec relative to the wave, it would take only 4½ hours for each air parcel to cover the 1000 km distance along which there is anticyclonic curvature. Suppose a particle passes the inflection point at 70 m/sec and from then on experiences a forward tangential acceleration $\left(\frac{dv}{dt}\right)_t = 2\Omega_z v_n$ on the anticyclonic bend. If v_n , the outward wind component normal to the isobars, reaches the high average value of 10 m/sec on the anticyclonic bend, the speed of the particles would increase at a rate of 10 m/sec per 3 hours, and at most by 15 m/sec during the whole travel from inflexion point to inflexion point. That increase in speed would thus only go one-fifth of the way from v_g to $2v_g$. This reasoning justifies the earlier assumption of the moderately super-geostrophic wind of

70 + 10 = 80 m/sec on the middle of the anticyclonic bend.

Another effect of the trans-isobaric wind component on the anticyclonic bend is also worth considering. A flow component across anticyclonic contours towards low pressure is usually synonomous with horizontal divergence of mass, and offers in that way a contribution to pressure fall (see equation (1)). The basic pattern in Figure 1 of horizontal divergence in westerly waves would thus in the levels of strongest westerlies show the divergence extending forward beyond the ridge of highest pressure. The implications of this divergence on the pressure ridges of the upper atmosphere for the storm development in the lower atmosphere will be considered on p. .

2b. THE FRONTOGENESIS.

Figure 10 illustrates three stages of frontogenesis, 24 hours apart, represented by simultaneous sea level and 850 mb maps. At the first map time the Hudson Bay cyclone is also shown. Its thermal structure is that of an old cyclone with the occluded front beginning to wrap around the center. The upper warm tongue, extending east and north of the Hudson Bay center from the warm air reservoir over the Atlantic, is shown clearly in the 850 mb isotherms. The pressure trough pointing southwards from the Hudson Bay cyclone is not of frontal nature as can be seen from the weak temperature gradient at 850 mb in that region. The same non-frontal trough continues up to the 300 mb level (Figure 9), where its orientation approaches NW-SE, and in those upper layers it actually extends out over the Atlantic producing a bend in the warm sector current. Such troughs always move slower than the air, and it is kinematically impossible for fronts to develop in them.

Historical continuity made it obvious that the cold front from the Hudson Bay cyclone had reached Florida by November 7, 1500Z, but the

850 mb map shows how the cold air after arriving over the Gulf States must have subsided and thereby effaced most of the frontal temperature contrast. The bundle of isotherms running along the northern part of the cold front bends westward over the Carolinas and northern Georgia and there marks the intersection of the 850 mb map with the tilting surface of subsidence. The 850 mb winds in that region blow across the isotherm bundle from cold to warm, but fail to produce any local fall in temperature because of the simultaneous sinking. The deviations from geostrophic flow are quite striking in that sinking air mass. While descending from the levels of strong west winds the air particles must retard and, in order to do so, they must move with a component towards high pressure, as shown on the 850 mb map of November 7, 1500Z. The same type of geostrophic departure is found on that day over the southeastern U.S. all the way up to 300 mb (Figure 9). On the following day (November 8) the geostrophic departures characteristic of the front side of moving anticyclones, can be seen on the 850 mb map over New England. In the rear of the moving anticyclone the opposite geostrophic deviation is observed. In that part the air is ascending and accelerating and must have a horizontal component towards low pressure. That phenomenon is actually part of the process of frontogenesis over the central U.S. which will next be considered.

Frontogenesis by horizontal advection operates when a field of deformation is maintained in a baroclinic air mass. Optimum efficiency in that process is achieved when the axis of dilatation of the field of deformation coincides with the direction of the isotherms. The streamlines in the col over the central U.S. on November 7, 1500Z, fulfill that condition fairly well. The mentioned geostrophic departure towards low pressure on the warm side of the cold also favors the transportation of the isotherms towards the axis of dilatation. As a result of these processes a surface front has formed on November 8, 1500Z, over the region previously occupied by the col, while frontogenesis is in progress over the Great Lakes region where the col has now arrived. A frontal wave has already formed over the state of Missouri and is represented in the pressure field by a small elongated low. During the following twenty-four hours the new cyclone deepens and moves along the front northeastward. A new field of deformation is active on November 9 along the cold front of that cyclone and helps maintain the frontal temperature contrast.

The described frontogenesis development conforms with the advective rules set forth by T. Bergeron (   ) and S. Petterssen (   ). We will here add a study of the dynamical conditions for isentropic up-gliding in the free atmosphere, which is an important part of the process of frontogenesis.

The rate of frontogenesis near the ground is increased considerably if the air of the lower part of the frontal zone is removed by up-gliding. The dynamical possibilities for that process are considered in Figure 11, which contains a profile across the zone of frontogenesis during its early stage on November 7, 1500Z. At that time no clear cut front was yet discernible on the surface map, but on the 850 mb map there is great crowding of isotherms between the cold air over North Platte and the foehn air over Dodge City. The foehn on the warm side of the col gives the frontogenesis a good start in the layers below the level of the continental divide, but the process goes on also higher up, as shown in the Rapid City isothermalcy between 600 and 560 mb.

The thermodynamics of up-gliding in the sloping frontal zone can be tested through an inspection of the inserted 293° dry-isentrope. It shows that a particle could be brought dry-adiabatically along the profile from near the ground in Oklahoma to the tropopause at 350 mb over Montana without being subject to stabilizing gravity effects. If we take into account that condensation would begin in such a particle at 700 mb the ascent from there on would follow the saturation isentrope of 282°, which climbs steeper than the dry-isentrope and likewise reaches the tropopause. Actually no single particle performs such far-reaching isentropic displacements inside the profile; but the isentropes still can be used as indicators of the direction of the component of stable up-gliding (see p.), which the particles may have in addition to their much stronger geostrophic component of motion normal to the profile.

A study of the values of the isentropic up-gliding (equation (11)),

$$v_\eta = \frac{v_x \frac{\partial v_g}{\partial x} + \frac{\partial v_g}{\partial t}}{2\Omega_z - \frac{\partial v_g}{\partial y}},$$

along the different sections of the inserted isentropes will reveal the dynamical possibilities for the frontogenesis. The denominator in the above expression can be determined uniquely from the data contained in the profile, whereas the numerator depends on derivatives of the geostrophic wind normal to the profile and derivatives in time. Let us consider first the denominator.

Along the lower part of the isentrope 293° the geostrophic shear $\frac{\partial v_g}{\partial y}$ is negative, and hence $2\Omega_z - \frac{\partial v_g}{\partial y}$ is large. Between the points of intersection of the 293° isentrope with the isovels of 20 m/sec and 10 m/sec the value of $2\Omega_z - \frac{\partial v_g}{\partial y}$ can be computed to be $1,6 \cdot 10^{-4}$ sec^{-1}, as indicated at the bottom of the profile. Farther up along the 293° isentrope $\frac{\partial v_g}{\partial y}$ changes sign and $2\Omega_z - \frac{\partial v_g}{\partial y}$ decreases despite the northward increase of $2\Omega_z$. In the baroclinic field between Rapid City and Glasgow $2\Omega_z - \frac{\partial v_g}{\partial y}$ has decreased to $0,8 \cdot 10^{-4}$ sec^{-1}. Still smaller values are found along the 282°

saturation isentrope, and a negative $2\Omega_z - \frac{\partial v_\eta}{\partial y}$ results in the section near Rapid City. The latter value indicates dynamic instability or, in other words, the condition of up-gliding without dynamic brake action. Actually only small volumes of saturated air (altostratus and cirrus) occur in the region under consideration during the early stage of frontogenesis, and friction of such air against the dry environment probably exerts enough of a brake action to preclude violent developments. The dry-adiabatic up-gliding thus still applies to the greater part of the baroclinic upper-tropospheric air.

The numerator in the expression for v_η can be judged from an inspection of the 500 mb map (in Figure 11). $v_x \frac{\partial v_\eta}{\partial x}$ measured on the map just north of Rapid City amounts to $20.2{,}6 \cdot 10^{-5}$ m sec^{-2} = $5{,}2 \cdot 10^{-4}$ m sec^{-2}. The large value of the term comes from the convergence of the 500 mb contours and that feature, in turn, is inherent in the structure of the large pressure trough to the west with its central area of weak pressure gradient bordering on strong pressure gradients to the south. The large value of $\frac{\partial v_\eta}{\partial x}$ is , of course, also corrobborated by measured wind velocities, which increase from 10 m/sec to 50 m/sec along the streamline from northern Wyoming to Green Bay, Wisconsin. Also the 300 mb map (Figure 9) shows the same convergence of contours a little farther north.

An evaluation of v_η at the 500 mb level just north of Rapid City gives:

$$v_\eta = \frac{5.2 \cdot 10^{-4}}{0.8 \cdot 10^{-4}} = 6.5 \text{ m/sec}$$

$\frac{\partial v_\eta}{\partial z}$ has here been neglected as insignificant in comparison with $v_x \frac{\partial v_\eta}{\partial x}$. The corresponding v_z would be about one hundredth of v_η , hence 6.5 cm/sec. Corresponding determinations of v_η lower down on the frontal slope result in smaller values, and consequently $\frac{\partial v_\eta}{\partial y}$ is positive. With $\frac{\partial v_x}{\partial \eta}$ and $\frac{\partial v_x}{\partial x}$ both positive in the frontal zone there is stretching in both η- and x-direction, so that frontogenesis progresses under optimum conditions.

In the upper troposphere south of the maximum westerlies $\frac{\partial v_g}{\partial \eta}$ assumes large values approaching those of $2\Omega_z$. In Figure 11 the measured $2\Omega_z - \frac{\partial v_g}{\partial \eta}$ at 300 mb over Oklahoma City is only $0.1 \cdot 10^{-4}$ sec^{-1}. $v_x \frac{\partial v_g}{\partial x} + \frac{\partial v_g}{\partial t}$ is, however, also small at that place (see Figure 9) so that no great v_η component results. Farther east near the Atlantic, where the anticyclonic isentropic shear is equally great and $v_x \frac{\partial v_g}{\partial x}$ large negative, v_η is observed to be large negative (directed towards high pressure) at all reporting upper wind stations. This is an example of the systematic non-geostrophic wind components at the "delta" of an upper jet stream derived in Figure 8. The horizontal convergence resulting in the southern half of the delta is instrumental in providing the pressure rise ahead of the moving high in the southeastern U.S. (Figure 10).

Figure 12 shows a profile through the zone of frontogenesis twenty-four hours later. The frontal slope has become steeper (1/50 in the lowest portion) and the frontal shear $-\frac{\partial v_x}{\partial \eta}$ is now characterized by a sharp 180° wind shift. Negative v_g values (NE wind) stronger than 10 m/sec now occur in the lower part of the cold wedge near the front. Applying the v_η-formula to particles in the NE current we find conditions set for isentropic down-gliding because the numerator $v_x \frac{\partial v_g}{\partial x} + \frac{\partial v_g}{\partial t}$ is now negative. A numerical estimate of the down-gliding along the isentrope of 281° near the cold edge of the frontal zone at 850 mb follows:

$$v_\eta = \frac{v_x \frac{\partial v_g}{\partial x} + \frac{\partial v_g}{\partial t}}{2\Omega_z - \frac{\partial v_g}{\partial \eta}} = \frac{-10 \cdot 1.4 \cdot 10^{-5} - 10^{-4}}{0.96 \cdot 10^{-4} - 0.1 \cdot 10^{-4}} = -2.8 \text{ m/sec}$$

The resulting isentropic descent traverses the frontal zone with a component from the cold to the warm side. This non-geostrophic component towards the frontal trough, together with the frictional flow component in the same direction, accounts for the sub-geostrophic displacement of warm fronts in general. In some cases the non-geostrophic component normal to the front may permit the cold wedge to advance against a moderate geostrophic component from warm to cold.

In the upper part of the frontal zone isentropic up-gliding continues on November 8 just as on November 7, as can be seen from the convergence of contours between Omaha and Bismarck on the inset 500 mb map. The same contour convergence is found right over Omaha on the 700 mb map (not reproduced). In the profile the 284° saturation isentrope approximately follows the warm edge of the frontal zone. $\frac{\partial v_\gamma}{\partial \eta}$ measured along that isentrope gives values greater than $2\Omega_z$ from the condensation level up to 700 mb. That lower portion of the frontal zone is thus dynamically unstable; while higher up, where the 284° isentrope turns parallel to the v_{g} - isovels, finite speeds of up-gliding can be determined. An estimate of the up-gliding on the saturation isentrope of 284° at the 500 mb level gives

$$v_\eta = \frac{23 \cdot 1{,}4 \cdot 10^{-5}}{(1{,}0 - 0{,}1)\,10^{-4}} = 3{,}6 \text{ m/sec}$$

Exploring the whole frontal zone for non-geostrophic isentropic motion we find the conditions for down-gliding limited upwards by the zero isovel while up-gliding begins above that line. Moreover, we find that the dry-isentropes indicate a down-gliding with a horizontal component across the front from cold to warm, while the up-gliding along saturation isentropes keeps almost parallel to the frontal slope. The front profile therefore must begin to bulge forward from cold to warm in the lower layers while remaining rather unchanged higher up. That is what happens when the apex of the frontal wave goes by, as will be discussed in connection with the maps in Figure 14.

The anticyclonic shear south of the westwind maximum has grown to the state of dynamic instability as shown in Figure 12 by means of the v_y differentiation along the 333° isentrope. Shears rather close to the instability limit also extend down to the 500 mb level and make possible the rather big and systematic wind component towards low pressure observed on the 500 mb map southeast of the jet stream.

Figure 13 shows a profile across the cold front from Dodge City (Kansas) to Maxwell Field (Alabama) on November 9, 1500Z. The corresponding sea level and 850 mb maps are to be found in Figure 10. The profile shows that strong horizontal temperature gradients have formed all the way to the top of the diagram. The frontogenesis process through horizontal advection has actually been operating through the whole troposphere (in Figure 14 the resulting frontal zone shows up well even at the 300 mb level). The inclination of the frontal zone is about 1/50 in the upper portion, and it is quasi-vertical near the ground, while an intermediate portion around 7-800 mb tilts only by 1/130. The general shape of the profile can be understood as the result of the bulging forward of the lower portion of the cold wedge after the passage of the frontal wave apex. The non-geostrophic down-gliding responsible for that process was found to be dynamically justified from the study of the front profile 24 hours earlier, $v_x \frac{\partial v_y}{\partial x} + \frac{\partial v_y}{\partial t}$ being negative in the lower portion of the cold wedge. The sea level map for November 9, 1500Z, (Figure 10) shows a positive $\frac{\partial v_g}{\partial x}$ along the whole cold front (the geostrophic wind component parallel to the front in the cold air is increasingly negative as we pass towards the negative X -direction), but the actual wind component, v_x, parallel to the front is about zero in the forward part of the cold wedge. $\frac{\partial v_g}{\partial z}$ also is small. Hence it follows that the isentropic down-gliding, v_n, should be insignificant except where $2\Omega_z - \frac{\partial v_g}{\partial n}$ is zero or negative. The profile in Figure 13 shows an almost perfect parallelism of v_g-isovels and dry-isentropes in the cold wedge, so that $\frac{\partial v_g}{\partial n}\theta = 0$. Therefore, no dynamic instability occurs inside the cold air, not even in the upper part where the frontal zone is quite steep. The only place for dynamic instability to occur inside the cold air is right at the quasi-vertical part of the frontal surface near the ground, where a real temperature discontinuity exists. The air volume involved is however too small to appear on a profile of the scale used in Figure 13. The release of the dynamic instability at the cold front is responsible for maintaining the downdraft, which is always observed in a strip of a few kilometers width following the front passage. This cold air downdraft is instrumental in giving the cold front a greater speed of displacement than would have been indicated by the geostrophic wind determined from the sea level pressure distribution. In Figure 10 the computed twelve-hour geostrophic displacement of the cold front is represented by a short arrow of 80 km length, while at the same place the preceding twelve-hour displacement was 210 km and the subsequent one 460 km. The geostrophic wind component normal to the front computed from the 850 mb map is big enough to account for the front displacement, and we must assume that air from that level enters the frontal downdraft and carries westerly momentum to the surface layer. In the layers above 850 mb the cold air current is cyclonically curved and hence sub-geostrophic. With increasing height the wind in the frontal zone also becomes more and more parallel to the frontal boundary, so that the front displacement is less there than at the ground.

The pre-frontal air in the profile in Figure 13 has a lapse rate slightly less than the saturation adiabatic, but with the existing horizontal temperature gradient saturation-adiabatic ascent is possible at the rather steep angle of 1/50, as shown by the sample saturation isentropes of 289° and 292°. The measured values of $2\Omega_z - \frac{\partial v_y}{\partial y}$ show a close approach to dynamic indifference and even some dynamic instability. A saturation-isentropic up-gliding is in order at 850 mb, as can be seen from the convergence of contours ($v_x \frac{\partial v_y}{\partial x} > 0$) in the warm current intersected by the profile. The same is true for the 700 mb map (not reproduced) but not for the 500 mb and 300 mb maps (Figure 9). The frontal up-gliding should therefore be confined to the layers under 500 mb. This is also verified by the Little Rock sounding, which goes up through the cold front rain but shows a 35 per cent relative humidity at 500 mb. Thunderheads growing up from the cloud mass of the cold front would, of course, go well beyond 500 mb, but such phenomena of "vertical instability" were not reported in the case under consideration. Intermittent, light pre-frontal rain, which was reported as far as 300 km ahead of the cold front, can quite well be accounted for by the saturation-isentropic up-gliding. In the warm season such up-gliding in the tropical air current may be sufficient to trigger thunderstorm formation, which in turn may develop pre-frontal squall lines.

2c. THE STRUCTURE OF THE MATURING FRONTAL CYCLONE.

Figure 14 presents the sea-level, 700 mb, 500 mb and 300 mb maps for November 10, 0300Z, depicting the structure of the maturning frontal cyclone. The amplitude of the frontal wave on the surface map has now increased considerably, and the cold air from the rear begins to encircle the cyclone center. It is clearly seen how this occlusion process has had its first start only at the ground, while the isotherm patterns of the upper maps still indicate an open wave of small amplitude. This shows that while the wave travels along the front the frontal slope increases to a maximum at the wave apex. At that point the smooth wave "breaks" and a relatively shallow cold outbreak fans out along the ground. The mechanism of that breaking of the smooth wave probably lies in the dynamic instability of the lower part of the frontal zone, described in its stage of inception in Figure 12 and continuing in the form of the frontal downdraft in Figure 13. During the process the cold front part near the cyclone center suffers frontolysis through cold air down-gliding. This is always noticeable in the surface map analysis, and, on November 10, 0300Z, it also shows up in a slackening of the frontal temperature gradient on the 700 mb map. Farther south, where the cold front passes through the frontogenetic field of deformation, the frontal temperature gradient remains rather strong. The strongest frontal temperature gradient is, however, found in the 500 mb level where the breaking of the frontal wave has not yet started. The 300 mb map also shows fairly strong temperature gradients across the pressure trough, which may justify the use of the term "front" even at that level. Particularly striking is the crowding of isotherms in the south-western corner of the map, probably an effect of frontogenesis in a 300 mb col in the unmapped area west of Mexico. The 300 mb map under consideration is just tangent to a tropopause depression over the cold tongue of tropospheric air in the west. The map intersects the tropopause along the zone of lowest temperature across Labrador and northern Ontario. The temperature maximum east of the Hudson Bay cyclone is stratospheric. The cold air to its north is tropospheric and has been brought from lower latitudes as part of the warm sector shown in Figure 9. On the 500 mb map, which is entirely tropospheric, the Hudson

Bay center has a cold core with warmer surroundings both to the north and south.

The pressure minimum of the frontal wave at the Great Lakes continues up to 700 mb with some westward tilt, but has already at that level shrunk so as to be a minor feature in the pressure field compared to the old stationary Hudson Bay minimum. But the young cyclone over the Great Lakes has all the potentialities of development inherent in the solenoid field concentrated along the frontal wave all through the troposphere. As a hydrostatic consequence of the wave shaped pattern of isotherms the upper current above the closed center is likewise wave shaped with an anticyclonic bend over the warm front area of up-gliding.

The frontal cyclone deepened at the rapid rate of 14 mb per 12 hours during November 10, and ended up as a storm center of 975 mb over northern Labrador on November 12. We will briefly consider the application of the various theories of upper air divergence which may claim to explain the deepening. Considering first the formation of the upper pressure crest which precedes the surface cyclone we will see the nature of the interplay between lower and upper layers. The fact that the upper pressure crest moves at a speed of only 9 m/sec in a current of 70-80 m/sec shows that the upper wave cannot be a free one. The timing and place of the first appearance of the pressure crest on the 300 mb map (November 9, 0300Z) make it likely that the upper crest formed by the upward motion connected with the beginning frontal up-gliding in the nascent frontal wave. Once the upper wave is established, upper divergence will be located in the area between the pre-existing upper pressure trough and the new upper pressure crest ahead of it. With the trough in NNE-SSW orientation and the pressure crest in NW-SE orientation the half wave length between them shortens northward and makes the upper divergence particularly strong in that region. That is then also the region where the rapid deepening of the frontal cyclone takes place. So far the dynamics involved in the deepening process conforms with the principles of Figure 1. With that basic pattern accepted, we must also admit the existence of the following modifying processes.

The upper pressure crest is continually being fed from below by the rising motion in the front half of the cyclone and therefore is forced to maintain the same slow speed of propagation as the surface vortex. When the curvature of the anticyclonic bend of upper isobars becomes sufficiently strong, the fast upper current is unable to follow the isobars in gradient wind fashion, and horizontal divergence must result on both sides of the upper crest line (see p.). That component of upper divergence modifies the model in Figure 1 in the sense of extending the pressure fall farther ahead of the surface center.

The upper air divergence of the Ryd-Scherhag theories (p.) is also superimposed on the divergence and convergence inherent in the wave pattern. It can best be judged by comparing the geostrophic wind at two successive inflexion points. The average geostrophic wind between the 30,000 foot and 29,000 foot contours at the inflexion point SW of the Great Lakes was 60 m/sec, and between the same contours at the inflexion point over Labrador 42 m/sec. By that measure it can be concluded that the Great Lakes cyclone was situated under a "delta" of the upper current, and that an upper divergence pattern in the style of Figure 8 would be superimposed. That upper pattern would tend to produce pressure fall in the northern half and pressure rise in the southern half of the delta. It may be counted in favor of this reasoning that the Great Lakes cyclone proceeded northeastward to northern Labrador and not along the 300 mb contours ahead of the storm which would have meant east-northeastward propagation.

The vagueness of this concluding discussion on cyclone displacecyclone problem even in synoptic situations selected for their simplicity. Further progress will have to depend on more penetrating studies into the dynamics of vortices and waves, including the long wave relationships which have scarcely been mentioned in this article.

* * *

The sample cyclone described above had its early development near the ground, where the frontal upgliding of the warm air and the downgliding of the cold were a direct consequence of the preceding frontogenesis. Such wave-cyclogenesis is typical for all the frontogenesis areas of the middle and high latitudes. Another type of cyclogenesis occurs at times independently of any pre-existing low level front. The cyclogenetic mechanism in that case seems to lie in the dynamic instability of the upper tropospheric westerlies, which leads to the meandering of the current and the subsequent formation of an upper cold low. Cyclogenesis of that kind is described through two synoptic examples in the following article by E. Palmén. In the one, November 4, 1946, the upper low did not arrive into any frontogenetic area, and did not extend down to the surface. In the other case, November 17-18, 1948, a frontal wave action can be discerned but, in contrast to the case of November 7-10, 1948 the wave motion started first in the upper troposphere and later extended to low levels.

The experience from the mentioned cases, together with many others, point out the two main processes: unstable frontal wave action and unstable meandering of upper tropospheric flow as the causes of the formation of extratropical cyclones.

Legend of Illustrations.

Fig. 1 Schematic model of cyclone with lower vortex and upper wave part. West to east vertical profile shows location of horizontal divergence and convergence of mass.

Fig. 2 Successive stages of development of a frontal wave to an occluded vortex.

Fig. 3 Advective formation of the thermal upper wave by the winds blowing relative to the moving pressure wave.

Fig. 4 The motion of the warm and cold air relative to the moving frontal wave.

Fig. 5 Successive ($1 \rightarrow 3$) degeneration of sinusoidal wave pattern caused by excessive anticyclonic vorticity on wave crest.

Fig. 6 Profile of isentropic sloping $\chi\eta$-plane and sample distribution of the isovels of v_g in $\chi\eta$-coordinates.

Fig. 7 Meridional profiles through a model of straight westerlies with quasi-stationary polar front (E. Palmén and C.W. Newton, 1948). Left: Dashed lines show isotherms, and solid lines isovels of zonal geostrophic wind. Right: Dashed lines show the field of dry-isentropes (degrees absolute), and the saturation-isentrope of 281° in the frontal zone. Solid lines represent the quantity $2\Omega_z - \frac{\partial v_g}{\partial \eta}$ in unit 10^{-4} sec^{-1}.

Fig. 8 Isentropic convergence (shaded) and divergence (unshaded) in the regions of jet stream confluence and delta.

Fig. 9 12-hourly sequence of 300 mb maps during November 7-10, 1948. r_i = radius of isobaric curvature, $r_{min} = -\frac{\partial \Phi}{\partial r}/\Omega_z^2 = 2v_g/\Omega_z$. Asterisk marks position of apex of frontal wave on sea-level map. Half barb 5 m/sec, full barb 10 m/sec, triangular barb 50 m/sec.

Fig. 10 24-hourly sequence of sea level and 850 mb maps showing advective frontogenesis and beginning cyclogenesis. Upper winds: Half barb 5 m/sec, full barb 10 m/sec, triangular barb 50 m/sec.

Fig. 11 Profile of early frontogenesis November 7, 1948, 1500Z, and part of the 500 mb map for the same time. Sample evaluations of $2\Omega_z - \frac{\partial v_g}{\partial \eta}$ below diagram. Upper winds: Half barb 5 m/sec, full barb 10 m/sec, triangular barb 50 m/sec.

Fig. 12 Profile of the warm front of the incipient wave on November 8, 1948, 1500Z, and part of the 500 mb map for the same time. Sample evaluations of $2\Omega_z - \frac{\partial v_g}{\partial \eta}$ below diagram. Upper winds: Half barb 5 m/sec, full barb 10 m/sec, triangular barb 50 m/sec.

Fig. 13 Profile of the cold front on November 9, 1948, 1500Z. Sample evaluations of $2\Omega_z - \frac{\partial v_g}{\partial \eta}$ below diagram. Upper winds: Half barb 5 m/sec, full barb 10 m/sec, triangular barb 50 m/sec.

Fig. 14 Structure of maturing frontal cyclone at sea-level, 850 mb, 700 mb, and 300 mb, on November 10, 1948, 0300Z. Upper winds: Half barb 5 m/sec, full barb 10 m/sec, triangular barb 50 m/sec.

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